

Signalling under theft risk: firm pricing with buyers, thieves, and aftermarket resale

Jincheng Li

Department of Economics, University College London, London, UK

ljctony123@gmail.com

Abstract. Smartphone theft has surged alongside growing refurbished aftermarkets, yet standard pricing models overlook that thieves can observe product quality while buyers cannot. This paper analyses how a durable goods firm sets its price when buyers are uninformed about theft-protection quality, but thieves are fully informed, and a stolen product's resale value rises with the original price. This paper develops a three-player signalling game (firm, buyer, thief) and solves for equilibrium. The durable goods firm bears a per-incident operational cost if theft occurs. The study finds that when this cost is sufficiently high, a separating equilibrium emerges: the high-protection firm charges a premium that credibly signals quality, and theft is deterred. When the cost is low, a pooling equilibrium prevails: both quality types set the same price, quality remains hidden, and low-protection products are stolen. Applying the Intuitive Criterion refines the separating outcome to a unique prediction. The model shows that a firm's own operational theft cost is the critical determinant of whether price can serve as a reliable quality signal, with implications for firm strategy and theft-liability policy.

Keywords: signalling game, theft economics, durable goods, secondary market

1. Introduction

Smartphone theft has risen sharply in the UK. In the first half of 2025, Vodafone and Three reported a 967% increase in theft incidents, with 42% of thefts concentrated in London and the SouthEast. Apple products account for 80% of stolen devices [1]. Concurrently, the global refurbished smartphone market has grown substantially: used phone sales rose by over 10% in 2022 and a further 6% in 2023, while new smartphone sales declined [2]. The resale market provides thieves with a lucrative channel: stolen devices can be resold at prices that strongly correlate with the original retail price.

This environment creates a threeway strategic problem. A smartphone firm knows its device's theftprotection quality (e.g., remote locking, parts serialisation, account locks), which determines a thief's economic return from stealing it. Thieves, through resale networks and repair data, can also infer this quality. Buyers, however, cannot directly observe it at the point of sale. This is a specific manifestation of product uncertainty—the difficulty buyers face in evaluating a product's true characteristics and, crucially, its future performance. Dimoka et al. conceptualised product uncertainty and examined its effects in online

markets¹, distinguishing it from seller uncertainty [3]. Their central insight is that buyers, deterred more by unknown product quality and future reliability than by seller's reputation, rely on observable signals when performance cannot be predicted. In smartphone markets, "performance" includes the risk that theft will immediately destroy or diminish the product's value. Buyers who cannot evaluate a device's theft-protection quality may thus be systematically disadvantaged relative to thieves, who possess superior information through resale networks.

Existing research does not adequately model the joint determination of pricing, buyer behaviour, and strategic theft under asymmetric information. Standard signalling games consider only one uninformed receiver [4, 5]. The economics of crime literature [6, 7] treats theft as a response to expected punishment or resale disruption but does not endogenise the firm's price as a signal of theft-protection quality. Threeplayer models either assume exogenous theft or focus on legal compliance rather than durable goods aftermarkets (e.g., [8, 9]). Thus, no framework captures the strategic interaction among a durable goods producer², a buyer, and a thief under asymmetric information, where the thief's payoff depends directly on the primarymarket price through a resale value function.

This study therefore develops and analyses a three-player signalling game that models the strategic interactions among a firm, a buyer, and a thief in a durable goods market with asymmetric information. Specifically, it asks under what conditions the firm's price can credibly signal theft-protection quality to the buyer, given that a higher price also increases the thief's incentive to steal via the resale market. The analysis identifies the role of the firm's perincident operational theft cost in separating versus pooling equilibria. When theft occurs, the firm bears a perincident operational cost—covering complaint handling, replacements, and reputational damage—and this cost makes its pricing decision strategic³: the firm internalises the theft risk and chooses a price not only to attract buyers but also to manage its expected theft losses. Thus, the firm's pricing decision must simultaneously attract buyers, deter thieves, and control its exposure to theft-related losses.

The study addresses four main questions:

- (1) Under what conditions does a separating equilibrium exist, in which the durable goods firm's price fully reveals its theft-protection quality to the buyer?
- (2) Under what conditions does a pooling equilibrium prevail, in which both quality types set the same price and quality remains hidden?
- (3) How does the firm's perincident operational cost of theft affect the existence and nature of these equilibria?
- (4) When multiple separating equilibria exist, which one survives the Intuitive Criterion refinement?

The remainder of the paper is structured as follows: Section 2 reviews the relevant literature on the economics of crime, signalling games, durable goods aftermarkets, and multiplayer gametheoretic models of theft. Section 3 presents the formal model. Section 4 characterises weak Perfect Bayesian Equilibria, deriving necessary conditions for separating and pooling equilibria. Section 5 applies the Intuitive Criterion to refine the equilibrium set. Section 6 concludes.

¹ Dimoka et al. distinguish product uncertainty from seller uncertainty and show that product uncertainty has a stronger negative effect on price premiums. Their context is online markets, but the logic applies to any setting where buyers cannot inspect quality before purchase [3].

² E.g., a smartphone firm.

³ In practice, such a cost may vary across firms depending on customer service policies, brand value, and insurance arrangements. The model treats the cost as an exogenous parameter, but it could be endogenised in extensions.

2. Literature review

There are four interrelated strands of literature that inform the threeplayer signalling game developed in this paper: the economics of crime and theft, signalling games and price as a quality signal, durable goods and aftermarkets, and multiplayer game-theoretic models of theft under asymmetric information.

2.1. The economics of crime and theft

The modern economic analysis of crime begins with Becker, who modelled criminal behaviour as a rational choice: potential offenders compare the expected gain from an illegal act with the expected cost, primarily determined by the probability and severity of punishment [6]. Ehrlich provides empirical support, showing a robust inverse relationship between crime rates and both the probability and severity of sanctions [10]. This framework underpins the thief's decision rule in the present model: theft occurs when the expected gain from resale exceeds the conversion cost (e.g., effort to bypass protection, risk of detection in resale networks).

Extending the cost side beyond law enforcement, Ayres and Levitt demonstrated that theft deterrence can operate through the disruption of resale channels rather than through higher arrest rates [7]. Their study of Lojack—a hidden tracking device for cars—found that the reduction in auto thefts is driven primarily by the breakdown of "chopshop" resale networks, not by increased apprehension of thieves. This insight is central to the present model: the theftprotection quality of goods in the primary market directly raises the thief's conversion cost, reducing the profitability of resale and thereby reducing theft rates.

Clarke introduces the concept of "hot products"—durable consumer goods that are disproportionately targeted for theft [11]. Correlates include high resale value, portability, and weak traceability; smartphones, particularly highend models, exemplify hot products. Theft is thus a calculated response to product characteristics and market conditions.

A parallel literature examines private protection against crime under asymmetric information. Hotte and Ypersele analysed how the observability of protection efforts affects crime deterrence, showing that observable protection can induce a diversion effect: shifting crime to less protected targets [12]. However, unobservable protection reduces criminals' expected payoff and generates positive externalities. Baumann et al. extended this by allowing property value to be private information of the victim [13]. They found that when property value is unobservable, the victim's protection choices can signal value to potential offenders, sometimes leading to suboptimal investment.

These models, however, focus on a twoplayer interaction (victim and offender). The present model adds a third player—the buyer—whose purchasing decision depends on price and inferred protection quality, introducing a market transaction that precedes the theft decision.

2.2. Signalling games and price as a quality signal

In markets where product quality is unobservable to buyers at the time of purchase, firms may use price to signal quality. Wolinsky developed a model in which price uniquely signals quality in a "fulfilled-expectations equilibrium", showing that the equilibrium price exceeds marginal cost, and the markup increases as consumer information becomes less precise [4]. Milgrom and Roberts analysed limit pricing as a signalling device: an incumbent firm sets a low price to signal low costs and deter entry by a betterinformed rival [5].

A limitation of these classic signalling models is that they consider only a single uninformed receiver. In many realworld markets, however, there are multiple receivers with different information and objectives. Farrell and Gibbons studied cheaptalk signalling with two audiences, showing that the presence of an additional receiver can alter the sender's incentive to communicate truthfully [14]. The present model takes a

different approach: the two receivers (buyer and thief) have asymmetric information—the buyer is uninformed about the product theft-protection quality, while the thief is fully informed, and the thief's payoff depends directly on the firm's price through the resale value function. The firm's price must therefore simultaneously convince the buyer to purchase and discourage the thief from stealing.

Cho and Kreps introduced the Intuitive Criterion as a refinement of Perfect Bayesian Equilibrium in signalling games [15]. It restricts the multiple equilibria supported by arbitrary off-path beliefs by requiring that if a deviation could be profitable for only one type, the receiver must assign probability one to that type. This refinement is essential below, as it eliminates implausible separating equilibria and yields a unique prediction for the hightype price.

2.3. Durable goods, aftermarket, and resale value

The link between primarymarket prices and secondarymarket values is well established in the durable goods literature. Waldman surveyed the evolution of durable goods theory, highlighting the role of aftermarkets in shaping firm pricing and product design [16]. Early models (for example, those developed by Bulow [17] and Conlisk et al. [18]) focused on time inconsistency and the Coase conjecture. Later work relaxed the assumption that new and used goods are perfect substitutes.

Hendel and Lizzeri show that adverse selection in the secondary market systematically discounts used goods relative to new ones, and that primary and resale prices are strongly linked [19]. This provides theoretical grounding for the assumption that the thief's resale value is increasing in the primarymarket price. Empirically, used smartphone prices comove with new device prices, and highend models retain value better.

More directly relevant, Galiani et al. analysed the interaction between crime and durable goods, finding that product durability can signal theft risk to consumers, and that theft externalities affect the firm's optimal durability choice [20]. Their model, however, treats theft as an exogenous probability rather than as a strategic response to price and protection. The present model endogenises the thief's decision, making the thief's payoff a function of both price and protection level.

Waldman [21], Esteban and Shum [22] examined how secondary markets affect primary market pricing in oligopoly settings. While those models focus on competition and market power, the present model abstracts from the level of competition to isolate the signalling problem, taking the aftermarket as a given feature: thieves can resell stolen devices at a price positively and exogenously correlated with the original purchase price.

2.4. Game-theoretical models of theft with asymmetric information

A small but growing literature applies game theory to theft and crime settings with multiple strategic players. These models typically fall into two categories.

First, twoplayer games between a potential victim and an offender, where the victim chooses a protection level (often unobservable) and the offender decides whether to steal. Key contributions include Hotte and Ypersele on observable versus unobservable protection [12] and Baumann et al. on endogenous observability when property value is private information [13]. These models capture the victimoffender interaction but abstract from any market transaction or consumer behaviour.

Second, threeplayer or inspection games that introduce a third party—such as a regulator, a court, or an intermediary. DariMattiacci et al. analysed a signalling game with a seller, a buyer, and a goodfaith purchaser, where theft was exogenous, and the thief was not a strategic player [8]. Christmann and Klein studied a corporate compliance inspection game with a firm, a regulator, and a potential criminal, focusing on liability

regimes rather than consumer markets [9]. These models incorporate three players, but they do not endogenise the thief's response to the firm's price, nor do they link theft incentives to the resale value of a durable good.

None of the existing gametheoretic frameworks combines: a firm that sets a price to signal unobservable quality to a buyer; a thief who observes quality and decides to steal based on the price-dependent resale value; and a firmspecific operational cost of theft that determines whether price can credibly separate types. The model developed below adds this missing structure.

Thus, the model fits into and enriches the literature by: first, extending the rationalchoice crime framework [6, 7] to include firm pricing as a strategic variable; second, adapting signalling games [4, 15] to two receivers with asymmetric information; and third, complementing existing threeplayer theft models [8] by making the thief pricesensitive and the firm's operational cost from theft the key determinant of equilibrium selection.

3. Model

The interaction among a seller (smartphone firm), a buyer, and a thief is modelled as a signalling game with one sender and two receivers.

Nature first chooses the device's theft-protection quality, $\theta \in \Theta = \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L > 0$, with prior distribution $\pi_H \in (0,1)$ ⁴. This prior is common knowledge. The sender, the firm (F), then observes θ (its type) and posts a price $p \in P$, where $P = [p_1, p_2]$ and $p_2 > p_1 > 0$. The buyer (B), as the third player, observes p but not θ and chooses $b \in A_B = \{0,1\}$, where $b = 0$ denotes "not buy" and $b = 1$ denotes "buy". The game ends if $b = 0$. If $b = 1$, additionally, the thief (T) also observes (p, θ) and chooses $s \in A_T = \{0,1\}$, where $s = 0$ denotes "not steal" and $s = 1$ denotes "steal".

Formally, the histories of the game are:

stage 1 (Nature): $h_1 = (\theta) \in H_1 := \Theta$,

stage 2 (Firm): $h_2 = (\theta, p) \in H_2 := \Theta P$,

stage 3 (Buyer): $h_3 = (\theta, p, b) \in H_3 := \Theta \times P \times A_B$,

and stage 4 (Thief): $h_4 = (\theta, p, 1, s) \in H_4 := \Theta \times P \times \{1\} \times A_T$,

where H_1 , H_2 , H_3 , and H_4 are the sets of all possible histories at each stage of the game.

The buyer's information set is $I_B(p) = \{(\theta_L, p), (\theta_H, p)\} \subset H_2$, with the partition $I_B = \{I_B(p) : p \in P\}$. This reflects that the buyer observes the posted price p but not the firm's type θ . By contrast, both the firm and the thief observe the full history and therefore have singleton information sets.

The players' ex post payoffs are as follows: The firm (F)'s payoff:

$$u_F = \begin{cases} p - c\theta, & \text{if } b = 1, \text{ and } s = 0, \\ p - c\theta - r, & \text{if } b = 1, \text{ and } s = 1, \\ -c\theta, & \text{otherwise,} \end{cases}$$

per-incident operational loss when theft occurs.

$$\text{The buyer (B)'s payoff: } u_B = \begin{cases} v - p, & \text{if } b = 1, \text{ and } s = 0, \\ T(\theta)(-p - l) + (1 - T(\theta))(v - p), & \text{if } b = 1, \text{ and } s = 1, \\ 0, & \text{otherwise.} \end{cases}$$

denotes the buyer's valuation, $l > 0$ represents the additional disutility from theft, and $T : \Theta \rightarrow [0,1]$ denotes the probability that theft leaves the buyer with a net loss. It is assumed that $T'(\theta) < 0$, so a higher level of protection reduces buyer risk ($T(\theta_L) > T(\theta_H)$).

⁴ "H" denotes the high-protection type θ_H , and "L" denotes the low-protection type θ_L .

The thief (T)'s payoff: $u_T = \begin{cases} A(p) - b\theta, & \text{if } b = 1, s = 1, \\ 0, & \text{otherwise,} \end{cases}$ where $b > 0$ denotes the marginal cost of overcoming protection, and $A : P \rightarrow [a_1, a_2]$ is the resale value function, with $a_1 < a_2$, $A'(p) > 0$, and $A(p_1) > 0$.

Importantly, each player's payoff is represented by a von Neumann-Morgenstern utility function of the firm's posted price and the receiver's actions.

The firm's type, θ represents the device's theft-protection quality, encompassing mechanisms such as remote traceability, regulatory blocking, parts serialisation, and account locks⁵. A higher θ raises the thief's conversion cost ($b\theta$), thereby making theft less profitable⁶, and reduces the buyer's exposure to loss since $T'(\theta) < 0$. At the same time, stronger protection increases the firm's cost of production, captured by $c\theta$. Importantly, θ is determined at the product-development stage and remains fixed when the retail price is set. The firm observes θ , and thieves can also infer it (through bypass knowledge, fencing networks, repair data, and resale patterns), whereas buyers cannot directly observe θ at the point of sale.

The buyer, with valuation v , accepts the price if her expected utility is non-negative. If theft occurs, the payoff combines two states: with probability $1 - T(\theta)$, the buyer retains the device and obtains $v - p$; with probability $T(\theta)$, the buyer incurs a loss $-p - l$ ⁷.

From the thief's perspective, the resale value function $A(p)$ increases with the primary-market price, reflecting the positive co-movement of new and used durable-goods prices. Theft is profitable only if the gain $A(p)$ exceeds the conversion cost $b\theta$.

Since θ is determined before pricing, $c\theta$ is a sunk cost for the firm regardless of whether a trade occurs. If the buyer accepts the price, the firm receives p ; if theft follows, it also incurs a per-incident operational loss $r > 0$ (e.g., complaint handling service and administrative costs). When r is sufficiently high, the firm prefers prices that both clear the market and avoid theft, thereby aligning the firm's incentives with those of the buyer. In equilibrium, this may give rise to separation, with the high type signalling that a purchase is safe. By contrast, when r is small, pooling prices may be sustained even though theft occurs in the low-type state.

4. Equilibria

The analysis adopts *Perfect Bayesian Equilibrium (PBE)* as the solution concept and proceeds in two steps. First, a set of weak PBEs is characterised, permitting unrestricted off-path beliefs. Second, the analysis refines this set using the Intuitive Criterion [15].

Each player's behavioural strategy is defined as a mapping from their information set to a probability distribution over available actions:

The firm's behavioural strategy: $\sigma_F : \Theta \rightarrow \Delta(P)$,

The buyer's behavioural strategy: $\sigma_B : P \rightarrow \Delta(A_B)$,

The thief's behavioural strategy: $\sigma_T : \Theta \times P \rightarrow \Delta(A_T)$.

⁵ Key sources include *Apple's Platform Security Guide* (Activation Lock), U.S. policy under the *Smartphone Anti-Theft Voluntary Commitment*, and UK evidence from the *Home Office Review* (2016).

⁶ For example, the U.S. CTIA Stolen Phone Checker makes blocked devices harder to activate or resell, reducing their realisable value.

⁷ Note that $T(\theta)$ should not be interpreted simply as the probability the phone is stolen since, by construction, the thief always obtains the resale value. Instead $T(\theta)$ is better understood as the probability that the buyer fails to recover value—for example, the probability that a refund is unsuccessful.

The buyer's belief system is defined as (Equation (1)):

$$\mu : P \rightarrow \Delta(\Theta), \mu(p) = (\mu_L(p), \mu_H(p)) \quad (1)$$

where $\mu_H(p)$ denotes the posterior probability that the firm is of a high type, conditional on observing price p . For tractability, the analysis is restricted to pure strategies. In this case, the behavioural strategy distributions degenerate into single actions: each type of firm posts a single price, the buyer either accepts or rejects deterministically, and the thief either steals or does not steal. This restriction greatly simplifies the characterisation of equilibria while preserving the general framework.⁸

A tie-breaking rule is imposed such that, when indifferent, the buyer chooses to buy and the thief chooses not to steal.

A weak Perfect Bayesian Equilibrium (wPBE) is defined as (Equation (2)):

$$\sigma = (\sigma_F, \sigma_B, \sigma_T; \mu) \quad (2)$$

Satisfying the following conditions:

(1) Sequential rationality.

For every price p , the buyer's action maximises her expected payoff given beliefs $\mu(p)$ and the thief's response. For every (θ, p) with $b = 1$, the thief's action is optimal. For each θ , the firm's pricing rule maximises expected payoff given strategies and beliefs.

(2) Bayesian updating on-path.

In a pure-strategy setting, the firm's behavioural strategy degenerates to a single price for each type. Instead of $\sigma_F(p|\theta)$, define the indicator function:

$$1\{p = p_\theta\} = \begin{cases} 1, & \text{if type } \theta \text{ posts price } p \\ 0, & \text{otherwise.} \end{cases}$$

For any p in the support of both types (Equation(3)):

$$\mu_H(p) = \frac{\pi_H 1\{p=p_H\}}{(1-\pi_H)1\{p=p_L\} + \pi_H 1\{p=p_H\}}, \mu_L(p) = 1 - \mu_H(p) \quad (3)$$

(3) Unrestricted off-path beliefs.

At prices not reached in equilibrium, the buyer's beliefs are arbitrary.

The analysis considers three subsets of wPBE, partitioned by the firm's strategy:

- 1) Pooling equilibria: both types post the same price, $\sigma_F(\theta_L) = \sigma_F(\theta_H)$.
- 2) Separating equilibria: the two types post distinct prices, $\sigma_F(\theta_L) \neq \sigma_F(\theta_H)$.
- 3) Hybrid equilibria: strategies that are neither fully pooling nor fully separating.

For tractability, the analysis focuses on pure-strategy pooling and separating equilibria.

The thief's payoff is increasing in the resale value $A(p)$. Since A is strictly increasing and differentiable in p , the inverse A^{-1} exists. Define the theft-attempt threshold for type θ by $\bar{p}_\theta := A^{-1}(b\theta)$ with $\bar{p}_{\theta_L} < \bar{p}_{\theta_H}$.

⁸ Because PBE is a refinement of wPBE, any pure-strategy PBE in this model is necessarily also a pure-strategy wPBE.

This partitions the price space P into three intervals: $i_0 := [p_1, \bar{p}_{\theta_L}]$; $i_1 := (\bar{p}_{\theta_L}, \bar{p}_{\theta_H}]$; $i_2 := (\bar{p}_{\theta_H}, p_2]$.

Assume $b\theta_L$ is large enough that $p_1 < \bar{p}_{\theta_L}$, and $b\theta_H$ is small enough that $p_2 > \bar{p}_{\theta_H}$ so that all three intervals are non-empty.

Proposition 4.1 (Thief's Best Responses): In any wpBE, for all $p \in i_0$, the thief chooses $s = 0$, regardless of the firm's type. For all $p \in i_1$, the thief chooses $s = 1$ if $\theta = \theta_L$ and $s = 0$ if $\theta = \theta_H$. For all $p \in i_2$, the thief chooses $s = 1$, regardless of the firm's type.⁹

This result is intuitive. Theft never occurs when resale incentives are too weak; it occurs only for the low-protection type in the intermediate price region i_1 ; and it always occurs once resale incentives are sufficiently strong for both types. The proposition follows directly from the fact that the thief is a fully informed receiver in the model.

In particular, attention is restricted to the non-trivial region where the buyer's valuation lies strictly between the two thresholds: $v \in (\bar{p}_{\theta_L}, \bar{p}_{\theta_H})$

Proposition 4.2 (No Acceptance above Value): If $p > v$, the buyer never accepts the price, regardless of posterior beliefs and the thief's strategy.¹⁰

If the buyer observes p , the highest possible expected payoff from accepting is obtained by setting $\mu_H(p)$ and the thief's strategy: $\max_{\mu_H(p), s} E(\mu_B(\theta, p, 1, s)p) = v - p^{11}$, which is strictly negative if $p > v$.

If $v < \bar{p}_{\theta_L}$, then since trade cannot occur at prices above v , then no separating equilibrium can exist. In this case, trade never induces theft (*Proposition 4.1*), and the firm has no incentive to signal. Since even the highest feasible contract price does not induce theft, signalling is unnecessary.

Finally, we assume a sufficiently small $T(\theta_L)$ to ensure the existence of some prices at which trade occurs with theft when $\mu_L(p) = 1$. Formally, this requires $T(\theta_L) < \frac{v - \bar{p}_{\theta_L}}{v + l}$, where $0 < \frac{v - \bar{p}_{\theta_L}}{v + l} < 1$, since $0 < v - \bar{p}_{\theta_L} < v + l$, given $v \in i_1$.¹²

With the strategy maps, belief updating, and the price partition (i_0, i_1, i_2) established, the analysis now focuses on equilibrium outcomes under the assumptions $v \in i_1$ and $T(\theta_L) < \frac{v - \bar{p}_{\theta_L}}{v + l}$. The discussion proceeds in three steps: the separating benchmark, where the price both deters theft and reveals quality; pooling in i_1 , where the buyer purchases, but theft occurs only under the low type; and the Intuitive Criterion refinement, which eliminate belief-dependent wpBE artefacts.

4.1. Necessary conditions for separating equilibria

Since the analysis already restricts attention to equilibria where the firm chooses pure strategies, consider now a separating pricing rule $p: \Theta \rightarrow P$ with $p(\theta_L) \neq p(\theta_H)$. Denote these prices by p_L and p_H . By construction, the buyer's on-path beliefs are: $\mu_L(p_L) = 1$, $\mu_H(p_H) = 1$.

⁹ See Appendix A1.

¹⁰ The buyer simply cannot receive positive payoff from trade if price is higher than perceived value.

¹¹ Note that there are only two types, the buyer's belief is fully determined by $\mu_H(p)$ (with $\mu_L(p) = 1 - \mu_H(p)$). Hence, the maximisation problem can be expressed solely in terms of $\mu_H(p)$.

Off-path beliefs remain unrestricted. The next step is to establish the necessary conditions for a separating equilibrium. These conditions eliminate prices, strategies, and beliefs that cannot sustain separation. Define the buyer's worst-case acceptance cap as (Equation(4)):

$$p^{wca} := v - T(\theta_L)(v + l) \quad (4)$$

Lemma 4.1.1: In any wPBE, the buyer accepts every price $p \in [p_1, p^{wca}]$, regardless of her beliefs.

For prices in i_0 , the thief never steals, so the buyer's payoff is simply $v - p > 0$. For prices in $\left(\bar{p}_{\theta_L}, p^{wca}\right]$ (given $T(\theta_L) < \frac{v - \bar{p}_{\theta_L}}{v + l}$, this interval is non-empty), if the firm is of type θ_L , theft occurs but the buyer's expected payoff remains nonnegative as long as $p \leq p^{wca}$. Since this bound already ensures acceptance under the worst-case belief $\mu_L(p) = 1$, acceptance follows under any posterior belief.

By *Lemma 4.1.1*, the low type faces a choice between two boundary prices: the theft-free boundary $\bar{p}_{\theta_L}$ at the edge of i_0 and the buyer's worst-case acceptance cap (Equation(4)), which incorporates the additional theft cost r .

Lemma 4.1.2: In any separating equilibrium, if $r > p^{wca} - \bar{p}_{\theta_L}$, then $p_L = \bar{p}_{\theta_L}$. If $r < p^{wca} - \bar{p}_{\theta_L}$, then $p_L = p^{wca}$. If $r = p^{wca} - \bar{p}_{\theta_L}$, both boundary prices may be chosen.¹³

This lemma shows that the low type's optimal separating price depends on the relative size of the theft-related operational loss r compared to the "gap" between the buyer's worst-case acceptance cap (Equation(4)) and the theft-free boundary $\bar{p}_{\theta_L}$. If r is large, the low type avoids the theft risk by setting $p_L = \bar{p}_{\theta_L}$. If r is small, the low type pushes the price up to p^{wca} , exploiting the buyer's willingness to accept despite potential theft.

Proposition 4.1.3 (Monotone Separation): In any separating equilibrium, $p_H > p_L$ ¹⁴.

Crucially, in any separating equilibrium, Incentive Compatibility (IC) requires that each type of the privately informed firm prefer its prescribed strategy over mimicking the other type [23].

For a separating profile, the incentive compatibility constraints are (Equation(5)):

$$(IC_H) : u_F(\theta_H, p_H, b, s) \geq u_F(\theta_H, p_L, b, s) \quad (IC_L) : u_F(\theta_L, p_L, b, s) \geq u_F(\theta_L, p_H, b, s) \quad (5)$$

Since $v \in i_1$ and any feasible equilibrium price must satisfy $p_H \leq v$ (*Proposition 4.2*)¹⁵, the high type never sets a price in i_2 . Hence, no theft occurs against θ_H on path, and $u_F(\theta_H, p_H, 1, 0) = p_H - c\theta_H$, $u_F(\theta_H, p_L, 1, 0) = p_L - c\theta_H$.

By *Proposition 4.1.3 (Monotone Separation)*, $p_H > p_L$, so (IC_H) holds automatically: $p_H - c\theta_H > p_L - c\theta_H$.

It therefore remains to verify only the low-type condition: $(IC_L) : u_F(\theta_L, p_L, b, s) \geq u_F(\theta_L, p_H, b, s)$.

¹² With $\mu_L(p) = 1$, the buyer accepts the price if and only if her expected payoff is nonnegative: $T(\theta_L)(-p-l) + (1-T(\theta_L))(v-p) = v - p - T(\theta_L)(v+l) \geq 0 \iff p \leq v - T(\theta_L)(v+l)$. Since $T(\theta_L) \in (0,1)$, it follows that $-l < v - T(\theta_L)(v+l) < v$. Theft further requires $p > \bar{p}_{\theta_L}$, so together these imply $T(\theta_L) < v - \bar{p}_{\theta_L} < v - \bar{p}_{\theta_L}l/v+1$.

¹³ See Appendix A2.

¹⁴ See Appendix A3.

Case 1: $r > p^{wca} - \bar{p}_{\theta_L}$.

By Lemma 4.1.2, the low type chooses $p_L = \bar{p}_{\theta_L}$. If the low type deviates to p_H , its payoff is $u_F(\theta_L, p_H) = p_H - c\theta_L - r$, whereas sticking to p_L yields $u_F(\theta_L, p_L, 1, 0) = \bar{p}_{\theta_L} - c\theta_L$ ¹⁶.

Thus, $u_F(\theta_L, p_H, 1, 1) - u_F(\theta_L, p_L, 1, 0) = p_H - \bar{p}_{\theta_L} - r$.

The low type's IC condition (Equation(5)) holds if and only if $p_H \leq \bar{p}_{\theta_L} + r$ ¹⁷.

Meanwhile, separation requires $p_H \geq p^{wca}$ ¹⁸, and trade requires $p_H < v$. Hence, the set of feasible high-type prices p_H is $[p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$.

Since in this case, $r > p^{wca} - \bar{p}_{\theta_L}$, this interval has more than one element.

Case 2: $r < p^{wca} - \bar{p}_{\theta_L}$.

By Lemma 4.1.2, the low type chooses $p_L = p^{wca}$. Suppose the IC condition (Equation(5)) holds: $u_F(\theta_L, p_L, 1, 1) \geq u_F(\theta_L, p_H, 1, 1)$.

At these prices, the payoffs are $u_F(\theta_L, p_L, 1, 1) = p^{wca} - c\theta_L - r$, $u_F(\theta_L, p_H, 1, 1) = p_H - c\theta_L - r$.

Thus, the IC condition (Equation(5)) requires $p_H - p^{wca} \leq 0$ ¹⁹.

However, Proposition 4.1.3 (Monotone Separation) requires $p_H > p^{wca}$. This strict inequality contradicts the IC condition (Equation(5)). Hence, no separating equilibrium exists when $r < v - T(\theta_L)(v + l) - \bar{p}_{\theta_L}$.

Case 3: $r = p^{wca} - \bar{p}_{\theta_L}$.

By Lemma 4.1.2, the low type is indifferent between $p_L = p^{wca}$ or $p_L = \bar{p}_{\theta_L}$.

(1) If $p_L = p^{wca}$, by Proposition 4.1.3 (Monotone Separation), the high type must set $p_H > p_L$. If the low type deviates to p_H , its payoff is $u_F(\theta_L, p_H, 1, 1) = p_H - c\theta_L - r$, whereas sticking to p_L yields $u_F(\theta_L, p_L, 1, 1) = p^{wca} - c\theta_L - r$.

Hence, $u_F(\theta_L, p_H, 1, 1) - u_F(\theta_L, p_L, 1, 1) = p_H - p^{wca} > 0$.

Thus, the IC condition (Equation(5)) fails. Therefore, $p_L = p^{wca}$ cannot support separation at $r = v - T(\theta_L)(v + l) - \bar{p}_{\theta_L}$.

(2) If $p_L = \bar{p}_{\theta_L}$, at p_L , no theft occurs for the low type; deviating to any $p_H \geq p^{wca}$ induces theft.

The payoffs are $u_F(\theta_L, p_H, 1, 1) = p_H - c\theta_L - r$, $u_F(\theta_L, p_L, 1, 0) = \bar{p}_{\theta_L} - c\theta_L$.

Thus, the IC condition (Equation(5)) requires $p_H - \bar{p}_{\theta_L} - r \leq 0$. Substituting $r = p^{wca} - \bar{p}_{\theta_L}$ gives $p_H - \bar{p}_{\theta_L} - r \leq 0 \iff p_H - p^{wca} \leq 0 \iff p_H \leq p^{wca}$.

Combined with the separating requirement $p_H \geq p^{wca}$, we obtain $p_H = p^{wca}$.

Therefore, under Case 3, the only separating candidate is $p_L = \bar{p}_{\theta_L}$, $p_H = p^{wca}$.

¹⁵ By the tie-breaking assumption, the buyer accepts when is indifferent.

¹⁶ By Proposition 4.1, when $\theta = \theta_L$: if $p_L = \bar{p}_{\theta_L}$, the thief chooses $s = 0$; if $p_L = p^{wca} > \bar{p}_{\theta_L}$, the thief chooses $s = 1$.

¹⁷ IC condition holds if $u_F(\theta_L, p_L, b, s) \geq u_F(\theta_L, p_H, b, s) \iff u_F(\theta_L, p_H, 1, 1) - u_F(\theta_L, p_L, 1, 0) \leq 0 \iff p_H - \bar{p}_{\theta_L} - r \leq 0 \iff p_H \leq \bar{p}_{\theta_L} + r$.

¹⁸ See the proof of proposition 4.1.3 for more details.

¹⁹ The IC condition requires $u_F(\theta_L, p_H, 1, 1) - u_F(\theta_L, p_L, 1, 1) \leq 0 \iff p_H - c\theta_L - r - (p^{wca} - c\theta_L - r) = p_H - p^{wca} \leq 0$.

Summary (Incentive Compatibility). Taken together, the three cases establish the necessary conditions under which a separating equilibrium can exist. When $r > p^{wca} - \bar{p}_{\theta_L}$, the low type chooses $p_L = \bar{p}_{\theta_L}$, while the high type must select a price from the non-degenerate interval, $p_H \in [p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$.

When $r < p^{wca} - \bar{p}_{\theta_L}$, separation cannot occur, since the IC condition (Equation(5)) and monotone separation are mutually inconsistent. In the boundary case when $r = p^{wca} - \bar{p}_{\theta_L}$, the low type is indifferent between the two boundary prices, but only $p_L = \bar{p}_{\theta_L}$ is sustainable; the high type is then forced to price exactly at $p_H = p^{wca}$.

Proposition 4.1.4 (Necessary conditions for the Existence of Separating Equilibria): A separating equilibrium exists only if $r \geq p^{wca} - \bar{p}_{\theta_L}$.

Proposition 4.1.4 reflects the idea that separation requires the low type to find imitation unprofitable. When r is sufficiently high, mimicking the high type's higher price exposes the low type to costly theft, so it prefers its own "safe" boundary price, $\bar{p}_{\theta_L}$, which avoids theft, while the high type can charge more without incurring theft. By contrast, if r is small, theft does not deter imitation, and price signal loses its signalling power. A sufficiently large per-incident theft cost for the firm is the economic force that ensures separation: it disciplines the low type and allows prices to reveal quality.

4.2. Necessary conditions for pooling equilibria

The authors now turn to the case where the firm chooses $p(\theta_L) = p(\theta_H) = p^*$. The buyer's posterior belief at p^* coincides with the prior (Equation(6)):

$$\mu_H(p^*) = \frac{\pi_H 1\{p^* = p_H\}}{(1-\pi_H)1\{p^* = p_L\} + \pi_H 1\{p^* = p_H\}} = \frac{\pi_H^* 1}{(1-\pi_H)^* 1 + \pi_H^* 1} = \pi_H, \quad \mu_L(p^*) = 1 - \pi_H \quad (6)$$

And off-path beliefs are unrestricted.

By *Lemma 4.1.1*, the buyer accepts any $p \in [p_1, p^{wca}]$ regardless of beliefs. Hence no pooling price $p^* < \bar{p}_{\theta_L}$ can be sustained: at $p^* \in i_0$, the buyer accepts and the thief does not steal for either type, so a unilateral deviation by either type to $\bar{p}_{\theta_L}$ is profitable. The firm therefore cannot pool below $\bar{p}_{\theta_L}$.

Lemma 4.2.1: In any pooling equilibrium, $p^* \geq p^{wca}$.²⁰

Pooling prices below the buyer's worst-case acceptance cap cannot be sustained. If all types set such a low price, the high type can profitably deviate upward to p^{wca} . At that deviation, the buyer accepts regardless of beliefs, and theft does not occur for the high type. This deviation yields strictly higher profits for the high type than the pooling payoff.

Lemma 4.2.2: In any pooling equilibrium, $p^* \geq \bar{p}_{\theta_L} + r$.²¹

At the boundary price $\bar{p}_{\theta_L}$, the low type just avoids incurring the theft cost r while still completing the sale. If the pooling price does not exceed this boundary by at least r , the low type strictly prefers to deviate downward to $\bar{p}_{\theta_L}$.

Lemma 4.2.3: In any pooling equilibrium, $p^* \geq \max\{\bar{p}_{\theta_L} + r, p^{wca}\}$

Proof. From *Lemma 4.2.1* and *Lemma 4.2.2*, the firm chooses p^* no less than $\bar{p}_{\theta_L} + r$ and p^{wca} .

Combining these two conditions yields the stated inequality $p^* \geq \max\{\bar{p}_{\theta_L} + r, p^{wca}\}$.

Define the buyer's on-path acceptance cap as (Equation(7)):

$$p^{OPA} := v - (1 - \pi_H)T(\theta_L)(v + l) \quad (7)$$

Lemma 4.2.4: The buyer accepts a pooling price p^* if and only if $p^* \leq p^{OPA}$.

Proof. By Lemma 4.2.3, any pooling price must satisfy $p^* \geq \max\{\bar{p}_{\theta_L} + r, p^{wca}\}$. This implies that under pooling, theft always occurs at the low type θ_L . At the same time, pooling requires $p^* < v$ (Proposition 4.2); otherwise, the buyer would reject and the high type could profitably deviate to p^{wca} . Thus, any pooling equilibrium price lies in i_1 , where theft occurs only at θ_L .

Given a pooling price p^* , the buyer's posterior beliefs are $\mu_L(p^*) = 1 - \pi_H$ and $\mu_H(p^*) = \pi_H$. Her expected payoff is $E(u_B(\theta, p^*, 1, s)) = v - p^* - (1 - \pi_H)T(\theta_L)(v + l)$.

The buyer accepts if and only if $E(u_B(\theta, p^*, 1, s)) \geq 0$, which rearranges to $p^* \leq v - (1 - \pi_H)T(\theta_L)(v + l) = p^{OPA}$.²²

Note that p^{OPA} is always strictly larger than p^{wca} (but strictly smaller than v), since $\pi_H \in (0, 1)$. Unlike the worst-case scenario, pooling beliefs assign a strictly positive prior probability to meeting the high-type, in which case no theft occurs; this possibility of being 'lucky' makes the buyer willing to tolerate a price above p^{wca} because they incorporate the chance of avoiding theft.

However, to complete the argument, it is also required to show that $p^* \leq p^{OPA}$ is explicitly implied by sequential rationality and consistency.

Lemma 4.2.5: In any pooling equilibrium, $p^* \leq p^{OPA}$.²³

Proposition 4.2.6 (Necessary Conditions for the Existence of Pooling Equilibria): A pooling equilibrium exists only if $r \leq p^{OPA} - \bar{p}_{\theta_L}$.

Proof. From Lemma 4.2.3, any pooling price must satisfy $p^* \geq \max\{\bar{p}_{\theta_L} + r, p^{wca}\}$.

From Lemma 4.2.5, any pooling price must also satisfy $p^* \leq p^{OPA}$.

Therefore, a pooling equilibrium can exist only if $p^* \in \left[\max\left\{ \bar{p}_{\theta_L} + r, p^{wca} \right\}, p^{OPA} \right]$, where the set $\left[\max\left\{ \bar{p}_{\theta_L} + r, p^{wca} \right\}, p^{OPA} \right]$ must be non-empty: $p^{OPA} \geq \max\left\{ \bar{p}_{\theta_L} + r, p^{wca} \right\}$.

Since $p^{OPA} > p^{wca}$, the binding restriction is typically $p^{OPA} \geq \bar{p}_{\theta_L} + r$, which is equivalent to $r \leq p^{OPA} - \bar{p}_{\theta_L}$.

Proposition 4.2.6 implies that pooling is feasible only if $r \leq p^{OPA} - \bar{p}_{\theta_L}$. Economically, pooling can only be sustained when the firm's per-incident theft cost r is not too large. At a pooling price, theft still occurs for the low type; to prevent it from deviating to the no-theft boundary $\bar{p}_{\theta_L}$, the pooling price must rise by at least r . However, the buyer accepts at most p^{OPA} . Once $r > p^{OPA} - \bar{p}_{\theta_L}$, no price can both remain acceptable to the buyer and deter the low type from deviating, so pooling breaks down.

²⁰ See Appendix A4.

²¹ See Appendix A5.

²² $E(u_B(\theta, p^*, 1, s)) \geq 0 \iff v - p^* - (1 - \pi_H)T(\theta_L)(v + l) \geq 0 \iff p^* \leq v - (1 - \pi_H)T(\theta_L)(v + l)$.

²³ See Appendix A6.

5. Equilibrium refinements: the intuitive criterion

This section presents more general results on the existence of weak *Perfect Bayesian Equilibria*, extending the necessary conditions derived in Section 4 to full existence results before constructing explicit wPBE profiles.

Proposition 5.1 (wPBE Existence): A separating wPBE exists if and only if $r \geq p^{wca} - \bar{p}_{\theta_L}$. A pooling wPBE exists if and only if $r \leq p^{OPA} - \bar{p}_{\theta_L}$.

Proposition 5.2 (wPBE Profiles):

(1) *Separating profiles:* The firm sets $p_L = \bar{p}_{\theta_L}$, $p_H \in [p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$. The buyer accepts both on-path prices with on-path beliefs $\mu_L(p_L) = 1$ and $\mu_H(p_H) = 1$. For any off-path p , the buyer accepts if $p \in [p1, p^{wca}]$ and rejects otherwise, supported by the off-path belief being $\mu_L(p) = 1$ (for simplicity). The thief's strategy follows *Proposition 4.1*.

(2) *Pooling profiles:* The firm sets $p_L = p_H = p^*$, where $p^* \in [\max\{\bar{p}_{\theta_L} + r, p^{wca}\}, p^{OPA}]$. At the pooling price p^* , the buyer accepts with posterior beliefs $\mu_L(p_L) = 1 - \pi_H$ and $\mu_H(p_H) = \pi_H$. For any off-path p , the buyer accepts if $p \in [p1, p^{wca}]$ and rejects otherwise, supported by off-path belief being $\mu_L(p) = 1$ (for simplicity). The thief's strategy follows *Proposition 4.1*.²⁴

These profiles meet all requirements of a weak Perfect Bayesian Equilibrium: sequential rationality holds for each player, on-path beliefs follow Bayes' rule, off-path beliefs deter profitable deviations, the firm's strategies satisfy the incentive compatibility constraints established earlier, and the thief's strategy follows *Proposition 4.1*.

However, wPBE as a solution concept admits multiple equilibria, some of which rely on implausible off-path beliefs. The next section addresses this issue by applying a refinement.

5.1. Intuitive criterion

The wPBE set relies heavily on off-path beliefs: pooling can persist under pessimistic beliefs, and separation can be sustained by beliefs that do not credit profitability high-type deviations. To sharpen predictions, we apply the Intuitive Criterion of Cho and Kreps [15]. As shown below, this refinement trims the separating set but leaves the pooling set intact.

Definition 5.1.1 (Cho and Kreps, 1987: Intuitive Criterion [15]): Let $(\sigma_F^*, \sigma_B^*, \sigma_T^*; \mu^*)$ be a weak Perfect Bayesian Equilibrium (wPBE). Consider any off-path price $p' \in P$ that is not played with positive probability under σ_F^* . The set of potentially profitable deviators at p' is defined as (Equation(8)):

$$D(p') \{ \theta : u_F(\theta, p', b, s^*(\theta, p')) > u_F(\theta, p^*(\theta), b^*(p^*(\theta)), s^*(\theta, p^*(\theta))) \} \quad (8)$$

where $s^*(\theta, p)$ is uniquely determined by *Proposition 4.1*.

The Intuitive Criterion requires that if there exists a type $\hat{\theta}$ such that: $\hat{\theta} \in D(p')$, i.e., type $\hat{\theta}$ could strictly benefit from deviating to p' , and for all other types $\theta \neq \hat{\theta}$, $u_F(\theta, p', b, s) \leq u_F(\theta, p^*(\theta), b^*(p^*(\theta)), s^*(\theta, p^*(\theta)))$ i.e., no type other than $\hat{\theta}$ could ever benefit from deviating to p' , then the buyer's belief after observing p' must satisfy: $\mu_{\hat{\theta}}(p') = 1$, $\mu_{\theta}(p') = 0 \forall \theta \neq \hat{\theta}$ meaning the buyer assigns probability one to type $\hat{\theta}$.

²⁴ See Appendix A7.

In this section, it is sufficient to focus on off-path beliefs for $p' \in (p^{wca}, v]$. By *Lemma 4.1.1*, the buyer accepts any price in $[p_1, p^{wca}]$ regardless of her beliefs. Hence, whether the Intuitive Criterion restricts beliefs on these prices is irrelevant: the buyer's action does not change, and neither type of firm has an altered incentive to deviate. For prices strictly above v , trade never occurs. The firm would not deviate to any $p' \in (v, p_2]$, since remaining at an equilibrium price guarantees a strictly positive payoff. Consequently, only deviations to off-path prices within $(p^{wca}, v]$ matter for determining whether a wPBE survives under the Intuitive Criterion.

5.2. Refining separating equilibria

Proposition 5.2.1 (Separating Equilibrium under the Intuitive Criterion): Assume $r \geq p^{wca} - \bar{p}_{\theta_L}$ so that separating wPBE exist. Under the Intuitive Criterion, the separating equilibrium is uniquely given by: The firm chooses $p_L = \bar{p}_{\theta_L}$, $p_H = \min\{\bar{p}_{\theta_L} + r, v\}$.

The buyer accepts both on-path prices and any off-path price $p \in (p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$, with $\mu_H(p') = 1$.

The thief's strategy follows *Proposition 4.1*.

Proof. Construct a wPBE candidate profile (for both cases below): The firm chooses $p_L = \bar{p}_{\theta_L}$, $p_H \in [p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$.

The buyer accepts both on-path prices with $\mu_L(p_L) = 1$ and $\mu_H(p_H) = 1$; and for all off-path p , the buyer accepts if $p \leq p^{wca}$, with unrestricted beliefs, except where the Intuitive Criterion requires otherwise.

The thief's strategy follows *Proposition 4.1*.

Now consider a deviation $p' \in (p^{wca}, v]$.²⁵

Case 1: $r > p^{wca} - \bar{p}_{\theta_L}$.

For $p' \in (p_H, \min\{\bar{p}_{\theta_L} + r, v\})$: in i_1 , the thief does not steal against θ_H . The high type's payoff at p' is maximised when the buyer accepts (note that $p' < v$) since $u_F(\theta_H, p', 1, 0) = p' - c\theta_H > u_F(\theta_H, p', 0, 0) = -c\theta_H$ and this payoff is strictly greater than that obtained under p_H , given $p' \in (p_H, \min\{\bar{p}_{\theta_L} + r, v\})$:

$$u_F(\theta_H, p', 1, 0) = p' - c\theta_H > p_H - c\theta_H = u_F(\theta_H, p_H, 1, 0)$$

So the high type strictly benefits from deviating to p' . Since the thief does steal against θ_L in i_1 , the low type's payoff at p' when the buyer accepts is:

$u_F(\theta_L, p', 1, 0) = p' - c\theta_L - r < \bar{p}_{\theta_L} - c\theta_L = u_F(\theta_L, p_L, 1, 0)$ since $p' - \bar{p}_{\theta_L} < r$ given $p' \in (p_H, \min\{\bar{p}_{\theta_L} + r, v\})$. If the buyer were to reject at p' , the deviation is trivially unprofitable. Thus, for any $p' \in (p_H, \min\{\bar{p}_{\theta_L} + r, v\})$, only the high type could gain. By the Intuitive Criterion, the buyer must set $\mu_H(p') = 1$ and accept. Therefore, any p_H strictly below $\min\left\{\bar{p}_{\theta_L} + r, v\right\}$ is unstable — the high type would profit from a small upward deviation.

Next, consider $p' \in (\min\{\bar{p}_{\theta_L} + r, v\}, p_2]$. When $\bar{p}_{\theta_L} + r > v$, the buyer rejects at p' , and p' cannot be a profitable deviation for either type, so the off-path beliefs remain unrestricted. When $\bar{p}_{\theta_L} + r < v$, both types could weakly benefit relative to their on-path payoffs; at p' and given that the theft only occurs at θ_L (*Proposition 4.1*), both types' payoffs are:

²⁵ Only deviations $p' \in (p^{wca}, v]$ are relevant for Intuitive Criterion (See the final paragraph before section 5.1).

The high type receives $u_F(\theta_H, p', 1, 0) = p' - c\theta_H > p_H - c\theta_H = u_F(\theta_H, p_H, 1, 0)$, the low type receives $u_F(\theta_L, p', 1, 1) = p' - c\theta_L - r > \bar{p}_{\theta_L} - c\theta_L = u_F(\theta_L, p_L, 1, 0)$, since $p' > \bar{p}_{\theta_L} + r$. (Note that the consideration of both types' payoffs at p' when the buyer chooses not to buy is trivial: if the payoff at p' is maximised when the buyer rejects, then a deviation to p' from the equilibrium price p_L or p_H can never be profitable.).

Therefore, the buyer's beliefs at $p' \in (\min\{\bar{p}_{\theta_L} + r, v\}, p_2]$ are unrestricted.

Finally, any $p' < p_H$ is also not profitable: if $p' < p_H$, the high type earns less than at p_H , while the low type would incur the theft cost r , which exceeds any profit from deviating — since $p_H \in \left[p^{wca}, \min\left\{\bar{p}_{\theta_L} + r, v\right\}\right]$, and $p' < p_H \leq \bar{p}_{\theta_L} + r$, while a profitable deviation for the low type requires $p' - \bar{p}_{\theta_L} > r$.

Thus, the refined separating equilibrium in Case 1 has $p_H = \min\{\bar{p}_{\theta_L} + r, v\}$, and $p_L = \bar{p}_{\theta_L}$.

Case 2: $r = p^{wca} - \bar{p}_{\theta_L}$.

In this case, $p_H = p^{wca}$. From arguments in Case 1, deviations profitable for the high type alone are confined to the interval $(p_H, \min\{\bar{p}_{\theta_L} + r, v\})$. Since $\min\{\bar{p}_{\theta_L} + r, v\}$ becomes $\min\{p^{wca}, v\} = p^{wca}$ the interval $(p_H, \min\{\bar{p}_{\theta_L} + r, v\})$ is empty, so the Intuitive Criterion does not apply in this case. The firm chooses $p_H = p^{wca}$ and $p_L = \bar{p}_{\theta_L}$ in PBE.

5.3. Refining pooling equilibria

Proposition 5.3.1 (Pooling Equilibrium under the Intuitive Criterion): Assume $r \leq p^{OPA} - \bar{p}_{\theta_L}$, so that pooling wPBE exist. Under the Intuitive Criterion, the pooling set is unchanged: the firm sets $p_L = p_H = p^* \in [\max\{\bar{p}_{\theta_L} + r, p^{wca}\}, p^{OPA}]$.

The buyer accepts p^* on path, with $\mu_L(p^*) = 1 - \pi_H$ and $\mu_H(p^*) = \pi_H$. For any off-path price p , the buyer accepts if $p \leq p^{wca}$, and otherwise rejects, with off-path beliefs unrestricted.

The thief's strategy follows *Proposition 4.1: Proof. As before, only deviations $p' \in (p^{wca}, v]$ are relevant for the Intuitive Criterion.*

Fix any $p' \in (p^{wca}, v]$.

If $p' > p^*$, then, if the buyer accepts, both types strictly benefit ²⁶: $u_F(\theta_H, p', 1, 0) = p' - c\theta_H > p^* - c\theta_H$, $u_F(\theta_L, p', 1, 1) = p' - c\theta_L - r > p^* - c\theta_L - r$, since in i_1 theft occurs only for θ_L . Thus, the deviation is jointly profitable, so the Intuitive Criterion does not restrict the off-path beliefs.

If $p' \in (p^{wca}, p^*)$, then even if the buyer accepts, both types' revenues fall relative to p^* at p' .²⁷

Thus, the Intuitive Criterion imposes no further restriction on the pooling wPBE because there is no deviation from which only a single type would benefit.

²⁶ Again, the consideration for both type's payoff at p' when buyer chooses not to buy is somehow trivial; since if the payoff at p' is maximised when the buyer rejects, then a deviation to p' can never be more profitable than remaining at the equilibrium prices.

²⁷ When $p^* = p^{wca}$, the hypothesis $p' \in (p^{wca}, p^*)$ is no longer valid. Since only $p' \in (p^{wca}, v]$ is relevant for the Intuitive Criterion, it suffices to only consider $p' > p^*$.

6. Conclusion

This paper develops a threeplayer signalling game (buyer, thief, firm) to analyse how asymmetric information and theft risk jointly shape a firm's pricing strategy in durable goods markets, where the firm bears a per-incident operational cost r when theft occurs. The analysis yields four main answers to the research questions posed in the introduction.

First, a separating equilibrium – where price fully reveals theft-protection quality—exists if and only if the firm's perincident theft cost is sufficiently high, specifically $r \geq p^{wca} - \bar{p}_{\theta_L}$.²⁸

The lowprotection firm cannot profitably mimic a high price, so the high-protection firm can charge a premium that credibly signals quality.

Second, when r is low ($r \leq p^{OPA} - \bar{p}_{\theta_L}$), pooling equilibria prevail.²⁹ Both types set the same price, quality remains hidden from the buyer, and theft occurs on lowtype products.

Third, the firm's operational theft cost r is the pivotal parameter, determining both which equilibrium occurs and the range of feasible prices.

Fourth, applying the Intuitive Criterion [15] refines the set of separating equilibria to a unique outcome: the high type sets $p_H = \min\{\bar{p}_{\theta_L} + r, v\}$, while the low type sets $p_L = \bar{p}_{\theta_L}$. Pooling equilibria survive the refinement unchanged.

This paper extends signalling theory to two receivers with asymmetric information and endogenises theft as a response to the firm's price. Unlike previous threeplayer models (e.g., [8, 9]), it shows that the firm's own operational cost of theft determines whether price can credibly signal quality.

For firms, the model explains why companies with high customer-service standards and strong brand reputation—which imply a high perincident theft cost r —can sustain price premiums for security features even when their products are frequently targeted by thieves. In that equilibrium, theft does not actually occur against the hightype device because thieves know that conversion costs exceed resale value. For policymakers, the model suggests that mandating stricter liability or mandatory compensation schemes—thereby raising r —could incentivise firms to signal theftprotection quality truthfully and potentially increase social welfare.

There are, however, some limitations. The analysis is restricted to a static, purestrategy setting with two quality types and a single representative buyer, which in particular rules out the monopolistic control of the firm over product quality and aftermarket design. The firm's protection level θ is exogenously given; the model does not address the firm's investment decision in theft protection. The secondary market is reduced to an exogenous increasing function $A(p)$, with no endogenous aftermarket price formation, and the thief faces only a conversion cost rather than legal punishment. These simplifications were necessary for tractability but limit the model's direct applicability to more complex environments.

Several extensions are promising. First, endogenising the firm's choice of θ would allow analysis of investment incentives under different liability regimes. Second, introducing dynamic, repeated interaction could examine how learning and reputation affect signalling over time. Third, empirical testing of the core prediction—that higher firmlevel theft costs correlate with greater price dispersion and lower theft incidence—

²⁸ The term $p^{wca} - \bar{p}_{\theta_L}$ can be interpreted as the minimum price premium the low type would need to compensate for the theft loss. When r is larger than this gap, mimicking is unprofitable.

²⁹ In the intermediate region where $p^{wca} - \bar{p}_{\theta_L} \leq r \leq p^{OPA} - \bar{p}_{\theta_L}$, both pooling and separating equilibria coexist. Raising r beyond the upper bound of this region would eliminate pooling entirely.

would provide valuable validation. Fourth, incorporating different levels of market competition could reveal how market structure interacts with signalling and theft deterrence.

In conclusion, this paper has shown that the credibility of price as a signal of theft protection quality depends critically on the firm's own operational cost of theft. By modelling the thief as an active, rational agent whose profit depends on the primary market price through resale value, the analysis provides a more complete understanding of pricing under the shadow of theft. The results suggest that firms and policymakers alike should pay attention not only to law enforcement and consumer information but also to the internal costs that shape firms' strategic choices in theft-prone markets.

References

- [1] Sweney, M. (2025, September 6). *UK phone retailers lock shop doors while trading to tackle rising thefts*. The Guardian. <https://www.theguardian.com/business/2025/sep/06/phone-retailers-lock-doors-tackle-rising-thefts>
- [2] GSMA (2025). *Rethinking mobile phones: The business case for circularity*. GSMA.
- [3] Dimoka, A., Hong, Y., & Pavlou, P. A. (2012). On product uncertainty in online markets: Theory and evidence. *MIS Quarterly*, 36(2), 395-426.
- [4] Wolinsky, A. (1983). Prices as signals of product quality. *Review of Economic Studies*, 50(4), 647-658.
- [5] Milgrom, P., & Roberts, J. (1982). Limit pricing and entry under incomplete information: An equilibrium analysis. *Econometrica*, 50(2), 443-459.
- [6] Becker, G.S. (1968). Crime and punishment: An economic approach. *Journal of Political Economy*, 76(2), 169-217.
- [7] Ayres, I., & Levitt, S.D. (1998). Measuring positive externalities from unobservable victim precaution: An empirical analysis of Lojack. *Quarterly Journal of Economics*, 113(1), 43-77.
- [8] Dari-Mattiacci, G., Guerriero, C., & Huang, Z. (2015). The Good-Faith Purchaser: Markets, Culture, and the Legal System. *Oxford Journal of Legal Studies*, 35(3), 543-574.
- [9] Christmann, R., & Klein, D. B. (2024). Game theory, compliance, and corporate criminal liability: Insights from a three player inspection game. *Decision Analytics Journal*, 11, 100431.
- [10] Ehrlich, I. (1973). Participation in illegitimate activities: A theoretical and empirical investigation. *Journal of Political Economy*, 81(3), 521-565.
- [11] Clarke, R.V. (1999). *Hot products: Understanding, anticipating and reducing demand for stolen goods*. Police Research Series Paper 112. London: Home Office.
- [12] Hotte, L., & Ypersele, T. V. (2008). Individual protection against property crime: Decomposing the effects of protection observability. *Canadian Journal of Economics*, 41(2), 537-563.
- [13] Baumann, F., Denter, P., & Friehe, T. (2019). Hide or show? Observability of private precautions against crime when property value is private information. *American Law and Economics Review*, 21(1), 209-245.
- [14] Farrell, J., & Gibbons, R. (1989). Cheap talk with two audiences. *American Economic Review*, 79(5), 1214-1223.
- [15] Cho, I.-K., & Kreps, D.M. (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2), 179-222.
- [16] Waldman, M. (2003). Durable goods theory for real world markets. *Journal of Economic Perspectives*, 17(1), 131-154.
- [17] Bulow, J. (1986). An economic theory of planned obsolescence. *Quarterly Journal of Economics*, 101(4), 729-750.
- [18] Conlisk, J., Gerstner, E., & Sobel, J. (1984). Cyclic pricing by a durable goods monopolist. *Quarterly Journal of Economics*, 99(3), 489-505.

- [19] Hendel, I., & Lizzeri, A. (1999). Interfering with secondary markets. *RAND Journal of Economics*, 30(1), 1–21.
- [20] Galiani, S., Jaitman, L., & Weinschelbaum, F. (2020). Crime and durable goods. *Journal of Economic Behavior and Organization*, 173, 146–163.
- [21] Waldman, M. (1996). Durable goods pricing when quality matters. *Journal of Business*, 69(4), 489–510.
- [22] Esteban, S., & Shum, M. (2007). Durable-goods oligopoly with secondary markets: The case of automobiles. *RAND Journal of Economics*, 38(2), 332–354.
- [23] Spence, M. (1973). Job market signaling. *Quarterly Journal of Economics*, 87(3), 355–374.

Appendix

Appendix A1. Proof of Proposition 4.1

Since the thief's information sets are singletons, sequential rationality implies that at any history $(\theta, p, 1) \in \Theta \times P \times \{1\}$ reached, the thief chooses a best reply:

$$s = 1 \iff u_T(\theta, p, 1, 1) = A(p) - b\theta > 0 \iff A(p) > b\theta \iff p > \bar{p}_\theta.$$

For $\theta = \theta_L$, the thief steals if and only if $p > \bar{p}_{\theta_L}$. At $p = \bar{p}_{\theta_L}$, $A(\bar{p}_{\theta_L}) - b\theta_L = 0$. By the tie-breaking rule, the thief chooses $s = 0$ at the boundary of i_0 .

For $\theta = \theta_H$, the thief steals if and only if $p > \bar{p}_{\theta_H}$. At $p = \bar{p}_{\theta_H}$, $A(\bar{p}_{\theta_H}) - b\theta_H = 0$. By the tie-breaking rule, the thief chooses $s = 0$ at the boundary of i_1 .

Using $\bar{p}_{\theta_L} < \bar{p}_{\theta_H}$ and the partition $i_0 := [p_1, \bar{p}_{\theta_L}]$, $i_1 := (\bar{p}_{\theta_L}, \bar{p}_{\theta_H}]$, $i_2 := (\bar{p}_{\theta_H}, p_2]$, $s = 0$ for both types when $p \in i_0$, $s = 1$ for θ_L and $s = 0$ for θ_H when $p \in i_1$, and $s = 1$ for both types when $p \in i_2$.

Appendix A2. Proof of Lemma 4.1.2

Lemma 4.1.1 implies that the buyer always accepts prices (including on-path price p_L) in $[p_1, p^{wca}]$. Now focus on the case when the buyer observes $p_L > p^{wca}$ and the thief chooses to steal at θ_L . On-path Bayesian updating gives $\mu_L(p_L) = 1$. The buyer's expected payoff from accepting the trade is (Equation (A1)):

$$E(u_B(\theta, p_L, 1, 1)) = (1 - T(\theta_L))(v - p_L) + T(\theta_L)(-p_L - l) = v - p_L - T(\theta_L)(v + l) \quad (A1)$$

The buyer rejects the offer p_L if and only if Equation (A2):

$$E(u_B(\theta, p_L, 1, 1)) = v - p_L - T(\theta_L)(v + l) < 0 \quad (A2)$$

which is equivalent to Equation (A3):

$$p_L > v - T(\theta_L)(v + l) = p^{wca} \quad (A3)$$

Hence, in a separating equilibrium, trade cannot occur when the low type sets $p_L > p^{wca}$. Such a price induces deviation: by *Lemma 4.1.1*, the firm can profitably switch to $p_L = \bar{p}_{\theta_L}$, where the buyer still accepts ($\bar{p}_{\theta_L} \in [p_1, p^{wca}]$) and no theft occurs ($\bar{p}_{\theta_L} \in i_0$), yielding the low type's payoff (Equation (A4)):

$$u_F = \bar{p}_{\theta_L} - c\theta_L > -c\theta_L \quad (\text{A4})$$

Thus, in any separating equilibrium, the low type always chooses at least $p_L = \bar{p}_{\theta_L}$. Moreover, for any on-path $p_L < \bar{p}_{\theta_L}$, the firm can profitably deviate to $p_L + \epsilon$ ($\epsilon > 0$ is small) and obtain a strictly higher payoff. Therefore only $p_L \geq \bar{p}_{\theta_L}$ can be sustained.

Now consider $p_L \in \left(\bar{p}_{\theta_L}, p^{wca} \right]$. In this region, the low type's payoff is Equation (A5):

$$u_F = p_L - c\theta_L - r \quad (\text{A5})$$

Since the payoff is increasing in p_L , any interior price is dominated by a slightly larger price³⁰. The choice reduces to the two boundaries ($p_L = \bar{p}_{\theta_L}$ or $p_L = p^{wca}$):

Payoff from p^{wca} : $u_F = p^{wca} - c\theta_L - r$,

Payoff from $\bar{p}_{\theta_L}$: $u_F = \bar{p}_{\theta_L} - c\theta_L$,

and the low type prefers p^{wca} to $\bar{p}_{\theta_L}$ if and only if Equation (A6):

$$p^{wca} - c\theta_L - r > \bar{p}_{\theta_L} - c\theta_L \iff r < p^{wca} - \bar{p}_{\theta_L} \quad (\text{A6})$$

Similarly, if $r > p^{wca} - \bar{p}_{\theta_L}$, the low type prefers $\bar{p}_{\theta_L}$. If equality holds, the low type is indifferent.

Appendix A3. Proof (by contradiction) of Proposition 4.1.3

Suppose instead that $p_H < p_L$. Recall from *Lemma 4.1.2* that the low type's separating price p_L must lie on the boundary, with its exact value depending on the theft cost r .

Now consider a deviation by the high type to Equation (A7):

$$p' = p^{wca} \quad (\text{A7})$$

By *Lemma 4.1.1*, the buyer accepts p' under any belief. Since $p' \in i_1$, the thief does not steal when the type is θ_H . Hence, the high type strictly increases its payoff by deviating from $p_H < p_L$ to $p' \geq p_L$, contradicting the assumption that $p_H < p_L$. Separation also rules out the case $p_H = p_L$. Therefore, in any separating equilibrium, it must hold that $p_H > p_L$.

³⁰ $p_L - c\theta_L - r + \epsilon > p_L - c\theta_L - r$, where $\epsilon > 0$ is small.

Appendix A4. Proof of Lemma 4.2.1

First, pooling prices in i_0 cannot be sustained. Next, consider any pooling price $p^* \in [\bar{p}_{\theta_L}, p^{wca}]$. If the high type deviates to $p(\theta_H) = p^{wca}$, the buyer accepts (regardless of belief) and the thief chooses not to steal under $\theta = \theta_H$. The high type's payoff in this deviation is $u_F = p^{wca} - c\theta_H$, which strictly exceeds the pooling payoff $p^* - c\theta_H$. Hence no pooling equilibrium can exist for such prices, and we must have $p^* \geq p^{wca}$.

Appendix A5. Proof (by contradiction) of Lemma 4.2.2

Suppose instead that $p^* < \bar{p}_{\theta_L} + r$ in a pooling equilibrium. By Lemma 4.2.1 we know $p^* > \bar{p}_{\theta_L}$, so that theft occurs at $\theta = \theta_L$. The low type's payoff under pooling is $u_F = p^* - c\theta_L - r$.

If the low type instead deviates to $\bar{p}_{\theta_L}$, no theft occurs and the buyer still accepts, yielding $u_F = \bar{p}_{\theta_L} - c\theta_L$.

The difference is $p^* - c\theta_L - r - (\bar{p}_{\theta_L} - c\theta_L) = p^* - \bar{p}_{\theta_L} - r$. Since $p^* < \bar{p}_{\theta_L} + r$, the difference is negative, meaning the low type strictly prefers deviation. This contradicts the notion of pooling equilibrium, so we require $p^* \geq \bar{p}_{\theta_L} + r$.

Appendix A6. Proof (by contradiction) of Proposition 4.2.5

Suppose that in a pooling equilibrium, $p^* > p^{OPA}$. By Lemma 4.2.4, the buyer rejects, and both types obtain $-c\theta$. Consider a deviation by the high type to $p' = p^{wca}$. By Lemma 4.1.1, the buyer accepts p' regardless. Moreover, $p' \in i_1$, so the thief does not steal against θ_H . Hence, such a deviation yields profit p' , a contradiction of the assumption of a pooling equilibrium.

Appendix A7. Proof of Proposition 5.1 and Proposition 5.2 (Sufficient and Necessary Conditions for wPBE)

The two sufficient conditions are examined in turn:

(1) A separating wPBE exists if $r \geq p^{wca} - \bar{p}_{\theta_L}$.

Assume $r \geq p^{wca} - \bar{p}_{\theta_L}$. Construct a profile such that:

Construct the separating profile of Proposition 5.2 (1)

First, the thief's strategy is already sequentially rational by construction (Proposition 4.1).

Second, the buyer's on-path beliefs are $\mu_L(p_L) = 1$ and $\mu_H(p_H) = 1$. This generates the expected payoff from trade: At $p_L = \bar{p}_{\theta_L}$, $E(u_B(\theta, p_L, 1, 0)) = u_B(\theta_L, p_L, 1, 0) = v - \bar{p}_{\theta_L} > 0$, where $s = 0$ at the low type since $p_L \in i_0$, and the acceptance is optimal for the buyer.

At $p_H \in [p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$, $E(u_B(\theta, p_L, 1, 0)) = u_B(\theta_L, p_L, 1, 0) = v - p_H > 0$, where $s = 0$ at the high type since $p_H \in i_1$, and the acceptance is optimal for the buyer.

For off-path prices $p' \in [p_1, p^{wca}]$, the acceptance is optimal with unrestricted beliefs (Lemma 4.1.1); for $p' > p^{wca}$, thief chooses to $s = 1$ since $p' > \bar{p}_{\theta_L}$; $\mu_L(p') = 1$ and $\mu_H(p') = 0$, and the rejection is the best response (Equation (A8)):

$$\begin{aligned} E(u_B(\theta, p', 1, 1)) &= u_B(\theta_L, p', 1, 1) = T(\theta_L)(-p' - l) + (1 - T(\theta_L))(v - p') \\ &= v - p' - T(\theta_L)(v + l) = p^{wca} - p' \end{aligned} \quad (A8)$$

which is strictly negative since $p' > p^{wca}$.

Moreover, the incentive compatibility holds:

The low type receives $u_F(\theta_L, p_L, 1, 0) = \bar{p}_{\theta_L} - c\theta_L$ at p_L , while receives $u_F(\theta_L, p_H, 1, 1) = p_H - c\theta_L - r$ at p_H . This implies (Equation (A9)):

$$u_F(\theta_L, p_L, 1, 0) - u_F(\theta_L, p_H, 1, 1) = \bar{p}_{\theta_L} - c\theta_L - (p_H - c\theta_L - r) = r - (p_H - p_L) \geq 0 \quad (A9)$$

since $r \geq p^{wca} - \bar{p}_{\theta_L}$, and $p_H \in [p^{wca}, \min\{\bar{p}_{\theta_L} + r, v\}]$; and the high type receives $u_F(\theta_H, p_H, 1, 0) = p_H - c\theta_H$ at p_H and $u(\theta_H, p_L, 1, 0) = p_L - c\theta_H$ at p_L , which implies (Equation (A10)):

$$u_F(\theta_H, p_H, 1, 0) = p_H - c\theta_H > p_L - c\theta_H = u(\theta_H, p_L, 1, 0) \quad (A10)$$

One can further show that deviation to any off-path p' is never profitable for either the high or low type, given the assumed profile.

The on-path beliefs are also consistent: they are derived from Bayes' rule.

The off-path beliefs are arbitrary by the definition of wpBE.

Since all players are sequentially rational, given the strategies and beliefs, and beliefs are consistent on-path, the constructed profile constitutes a separating wpBE, given $r \geq p^{wca} - \bar{p}_{\theta_L}$.

(2) A pooling equilibrium exists if $r \leq p^{OPA} - \bar{p}_{\theta_L}$.

Assume $r \leq p^{OPA} - \bar{p}_{\theta_L}$. Consider a profile such that:

Consider the pooling profile of *Proposition 5.2* (2)

First, the thief's strategy is already sequentially rational by construction (*Proposition 4.1*).

Second, the buyer's on-path beliefs are $\mu_L(p^*) = 1 - \pi_H$ and $\mu_H(p^*) = \pi_H$. This generates the expected payoff from trade (Equation (A11)):

$$\begin{aligned} E(u_B(\theta, p^*, 1, s)) &= (1 - \pi_H)u_B(\theta_L, p^*, 1, 1) + \pi_H u_B(\theta_H, p^*, 1, 0) \\ &= (1 - \pi_H)\{T(\theta_L)(-p^* - l) + (1 - T(\theta_L))(v - p^*)\} + \pi_H(v - p^*) \end{aligned} \quad (A11)$$

where thief chooses $s = 1$ only at the low type. The acceptance is optimal (i.e., $E(u_B(\theta, p^*, 1, s)) > 0$) when $p^* \in [\max\{\bar{p}_{\theta_L} + r, p^{wca}\}, p^{OPA}]$.

For off-path prices $p' \in [p_1, p^{wca}]$, the acceptance is optimal with unrestricted beliefs (*Lemma 4.1.1*); for $p' > p^{wca}$, thief chooses to $s = 1$ since $p' > \bar{p}_{\theta_L}$; $\mu_L(p') = 1$ and $\mu_H(p') = 0$, and the rejection is the best response.

Given beliefs and the buyer and the thief's strategies, one can further show that no deviation is profitable

for the firm at $p^* \in [\max\{\bar{p}_{\theta_L} + r, p^{wca}\}, p^{OPA}]$ for either type.

The on-path beliefs are also consistent: they are derived from Bayes' rule.

The off-path beliefs are arbitrary by the definition of WPBE.